# Model for Fitting Spin Vertical and Horizontal Resonance in AGS Run05

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**Abstract**

Firstly test the model using the IUCF 1998 experiment data. Expand the model after consider the AGS realistic case.

# 1 Test of the model

## 1.1 Resonance from the Exp.

This idea comes from an article : *Spin Flipping in the Presence of a Full Siberian Snake PRL Oct.5 1998.* Using an rf solenoid magnet to induce the depolarizing resonance while keeping the spin tune constant, the frequency $f_r$ is given by

$$f_r = f_c(k \pm \nu_s) \tag{1}$$

Here $f_c$ is the revolution frequency, and $k$ is an integer. Slowly sweeping the rf magnet's frequency through $f_r$ can flip the spin. The Froissart-Stora equation gives the ratio of the final polarization $P_f$ and the initial polarization $P_i$ :

$$P_f = P_i(2e^{\frac{-(\pi W)^2}{4\frac{\Delta f}{\Delta t}}} - 1) \tag{2}$$

$W$ is the resonance width in Hz, and $\frac{\Delta f}{\Delta t}$ is the resonance crossing rate, where $\Delta f$ is the frequency range during the ram time $\Delta t$. Compared to the original Froissart-Stora equation which is expressed by the resonance strength $\epsilon$ :

$$P_f = P_i(2e^{\frac{-\pi|\epsilon|^2}{2\alpha}} - 1) \tag{3}$$

Here $\epsilon$ is the resonance strength and $\alpha$ is the resonance crossing rate. There is a correlation between the $W$ and $\epsilon$. Therefore, the resonance strength can be gotten from the experiment data. Below is the deduce from equation (3) to equation (4).

$$P_f \quad = \quad P_i(2e^{\frac{-(\pi W)^2}{4\frac{\Delta f}{\Delta t}}} - 1)$$

$$
\begin{aligned}
&= P_i(2e^{\frac{-(\pi\frac{W}{f_c})^2}{4\frac{\Delta f}{f_c}\frac{1}{f_c\Delta t}}} - 1) \\
&= P_i(2e^{\frac{-(\pi\frac{W}{2f_c})^2}{\frac{\Delta f}{f_c}\frac{1}{\frac{w_c}{2\pi}\Delta t}}} - 1) \\
&= P_i(2e^{\frac{-\pi^2(\frac{W}{2f_c})^2}{2\pi\frac{\Delta f}{f_c}\frac{1}{\Delta\theta}}} - 1) \\
&= P_i(2e^{\frac{-\pi(\frac{W}{2f_c})^2}{2\frac{\frac{\Delta f}{f_c}}{\Delta\theta}}} - 1)
\end{aligned}
$$

Therefore, the resonance strength can be calculated from the resonance width and revolution frequency :

$$|\epsilon| = \frac{W}{2f_c} \tag{4}$$

$$\alpha = \frac{\frac{\Delta f}{f_c}}{\Delta\theta} \tag{5}$$

The experiment result shows the resonance strength is $\epsilon$ 0.000206. At the same time, the rf solenoid's amplitude was set at 6kV corresponding to an $\int B.dl = 1.6T$. Then the resonance strength $\epsilon$ also can be given by ( *"Comment on ..." by M.Bai et. PRST 8,09001 2005*) :

$$|\epsilon| = \frac{\phi}{2\pi} = \frac{1+G}{4\pi}\frac{B_{osc}L}{B\rho} \tag{6}$$

The resonance coming from this calculation is $\epsilon$ 0.000245, which agrees with that from the experiment data. Next, I will substitute this resonance strength into the model and compare the calculated polarization to the experiment one. If they agree to each other, it proves the model is correct.

## 1.2 Testing

This model comes from the book : *Spin Dynamics and Snakes in Synchrotrons, S.Y.Lee.* The average polarization around a resonance region is given by:

$$P_f = P_i\frac{|K - G\gamma|}{\sqrt{(K - G\gamma)^2 + |\epsilon|^2}} \tag{7}$$

where $K$ is the resonance position, $G\gamma$ is the spin tune and $|\epsilon|$ is the resonance strength. From this equation, it shows that $|\epsilon|$ plays the role of the resonance width. At a distance $G\gamma - K = \pm|\epsilon|$ away from resonance, polarization is reduced by a factor of $\frac{1}{\sqrt{2}}$. In IUCF 1998 experiment, they kept the $G\gamma$ constant while changed the resonance position $K$ by the rf solenoid. Table1 shows the resonance position and measured polarization correspondingly. Substitute the resonance position into the model and select the resonance strength $\epsilon$ 0.000245, the calculated polarization is also shown on the table1. Fig1 gives the plot.

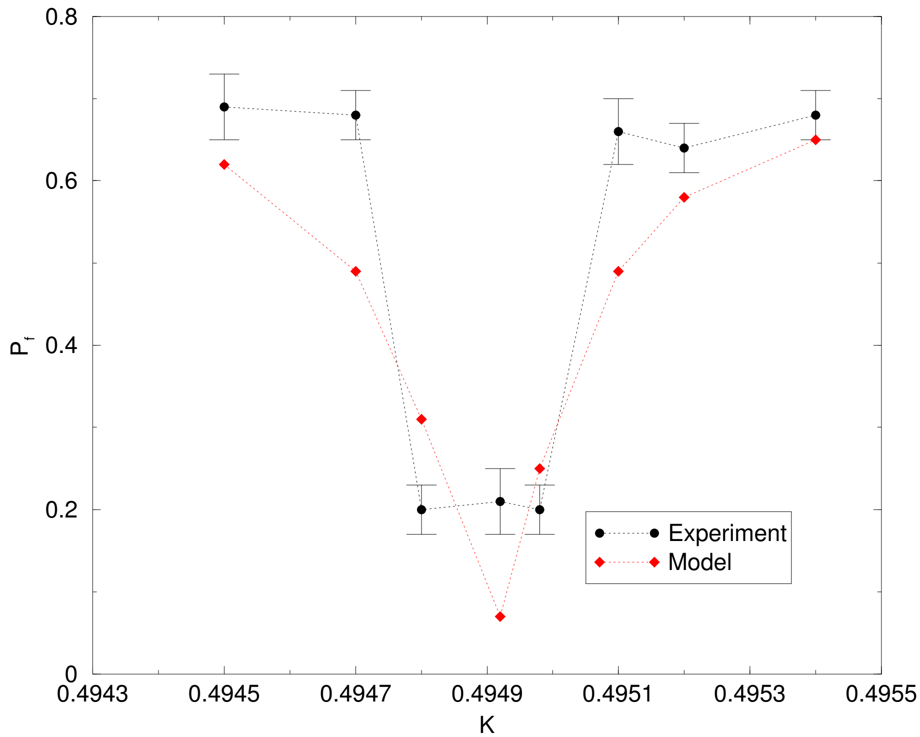


Figure 1: Black circles are the experiment data with the error bar, red ones are from the model using the resonance strength given by the experiment.

### 1.3 Conclusion

The plot shows the model and the experiment data do not agree well. The later application is based on the model with much more care about the realistic beam status.

## 2 Application on the experiments

In AGS 2005 Run, we did the B-field scan,which changed the spin tune $G\gamma$ instead of resonance position $K$. Use this model directly to fit the experiment data, it shows a big resonance strength from the width, which couldn't be. Here I give one of plots that shows B-field scan at $G\gamma = 38 + \nu_v$. Fig2 includes both the experiment data and the result from the model directly with different given resonance strength. They show the resonance strength can get to the order of $10^{-1}$, which is not reasonable for the real case. However, the experiment data does show such wide width, therefore, there must be some other reasons that cause the width wider. Consider the real beam distribution and tune spread,

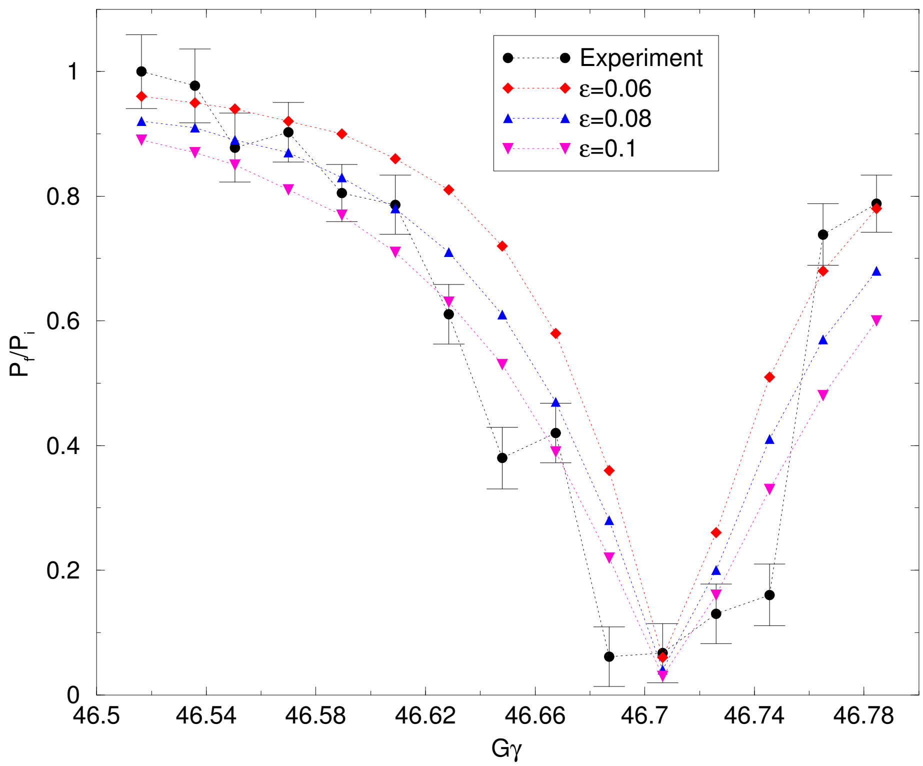


Figure 2: Black circles are the experiment data with the error bar, different colors for different resonance strength

the model can be modified as below:

$$
\begin{aligned}
P_f &= P_i \int_{-\infty}^{\infty}\int_0^{\infty} \frac{|K-G\gamma|}{\sqrt{(K-G\gamma)^2+\epsilon_0^2\frac{I}{I_0}}}\frac{1}{2I_0}e^{-\frac{I}{2I_0}}\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{(K-K_0)^2}{2\sigma^2}}dIdK \\
&= P_i \int_{-\infty}^{\infty}\int_0^{\infty} \frac{|(K_0+x_t)-G\gamma|}{\sqrt{((K_0+x_t)-G\gamma)^2+\epsilon_0^2\frac{I}{I_0}}}\frac{1}{2I_0}e^{-\frac{I}{2I_0}}\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{x_t^2}{2\sigma^2}}dIdx_t \\
&= P_i \int_{-\infty}^{\infty}\sqrt{\frac{\pi}{2}}\frac{|(K_0+x_t)-G\gamma|}{\epsilon_0}e^{\frac{((K_0+x_t)-G\gamma)^2}{2\epsilon_0^2}}(1-Erf[\frac{|(K_0+x_t)-G\gamma|}{\sqrt{2}\epsilon_0}]) \\
&\quad \frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{x_t^2}{2\sigma^2}}dx_t \qquad (8)
\end{aligned}
$$

where $I_0$ is the rms emittance of the beam,$\sigma$ is the rms tune spread,$K_0$ is the ideal resonance location. This equation includes two integration. The integration from 0 to $\infty$ means the beam has a Gaussian distribution, which can be gotten analytically. while the other one coming from the tune spread can not be. Leaving the second integration alone, The current work is trying to fit the experiment data using the above equation (8) numerically. In order to make the work much easier, some approximation was done, which is shown below.

For error function, it has the Rational Approximation ($0 \leq z < \infty$)

$$Erf(z) = 1 - (a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5) e^{-z^2} + O(z) \tag{9}$$

Here

$$\begin{aligned} t &= \frac{1}{1+pz} \\ |O(z)| &\leq 1.5 \times 10^{-7} \end{aligned} \tag{10}$$

and

$$a_1 = 0.254829592, a_2 = -0.284496736, a_3 = 1.421413741 \tag{11}$$

$$a_4 = -1.453152027, a_5 = 1.061405429, p = 0.3275911 \tag{12}$$

Here $z = \frac{|(K_0+x_t)-G\gamma|}{\sqrt{2}\epsilon_0}$. Because we alway have $0 \leq \frac{|(K_0+x_t)-G\gamma|}{\sqrt{2}\epsilon_0} < \infty$, the error function in equation (2) can be expanded by the equation (3). Then equation (2) becomes :

$$\begin{aligned} P_f &= P_i \int_{-\infty}^{\infty} \sqrt{\frac{\pi}{2}} \frac{|(K_0+x_t)-G\gamma|}{\epsilon_0} e^{\frac{((K_0+x_t)-G\gamma)^2}{2\epsilon_0^2}} \\ &\quad (1 - 1 + (a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5) e^{-\frac{((K_0+x_t)-G\gamma)^2}{2\epsilon_0^2}}) \\ &\quad \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x_t^2}{2\sigma^2}} dx_t \\ &= P_i \int_{-\infty}^{\infty} \sqrt{\frac{\pi}{2}} \frac{|(K_0+x_t)-G\gamma|}{\epsilon_0} \\ &\quad e^{\frac{((K_0+x_t)-G\gamma)^2}{2\epsilon_0^2}} (a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5) e^{-\frac{((K_0+x_t)-G\gamma)^2}{2\epsilon_0^2}} \\ &\quad \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x_t^2}{2\sigma^2}} dx_t \\ &= P_i \int_{-\infty}^{\infty} \sqrt{\frac{\pi}{2}} \frac{|(K_0+x_t)-G\gamma|}{\epsilon_0} (a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5) \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x_t^2}{2\sigma^2}} dx_t \end{aligned}$$

which has :

$$t = \frac{1}{1 + p\frac{|(K_0+x_t)-G\gamma|}{\sqrt{2}\epsilon_0}}$$

Theoretically,there are four parameters to be needed for fitting : $P_i$,$K_0$,$\sigma$,$\epsilon$ if we want get the best fitting result. There is something strange when I did it. For each different initialization, the fitted parameters are different totally. To simply the problem, I did the $\sigma$ scan by setting $P_i = -52.4$,$K_0 = 46.71$, which is shown in Fig3. For every $\chi^2$ minimum output, the resonance strength is approximately 0.08 with a little bit width expansion. In order to make clear

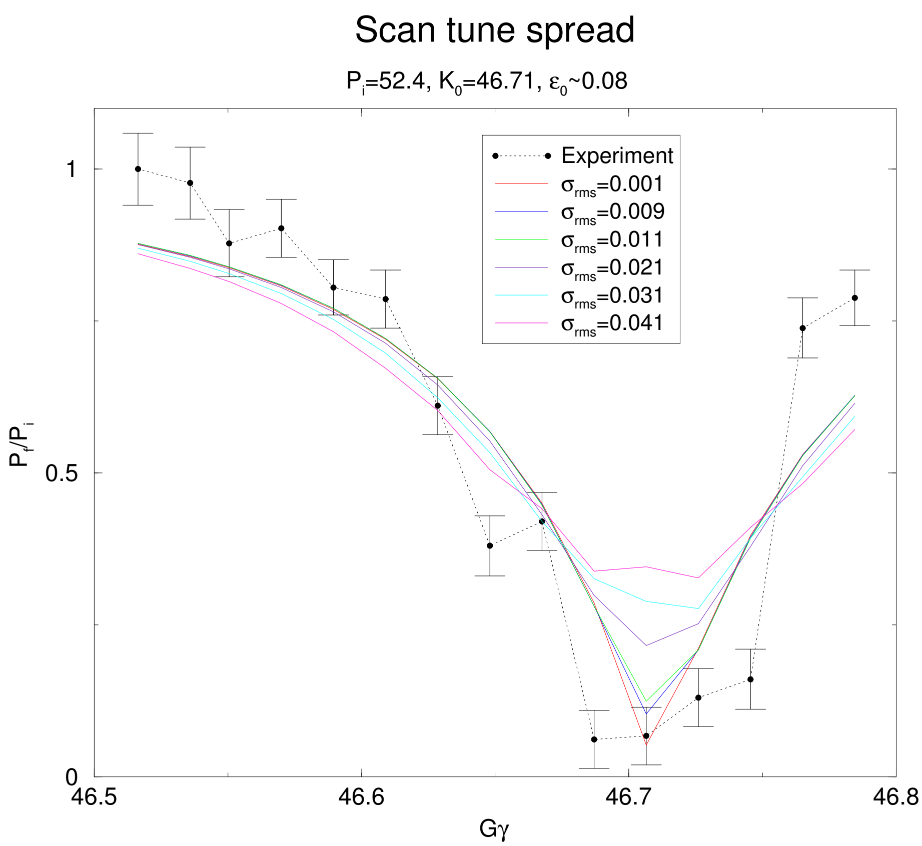


Figure 3: rms tune spread scan

that tune spread will widen the resonance strength, I also give the plot showing that phnomena in Fig4.

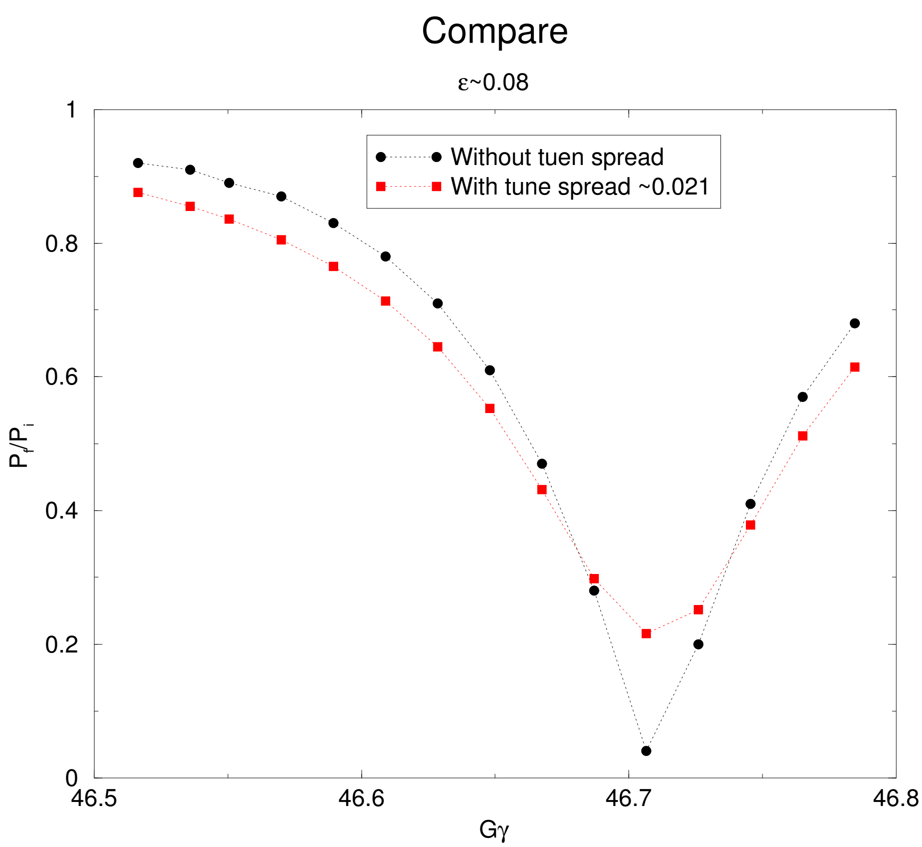


Figure 4: Black color shows no tune spread, and rea one shows with rms tune spread 0.021. Both of them have the same resonance strength